Enrollment No. ______________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION APRIL – 2017

M. Sc. Mathematics

16PMTCC08 - REAL ANALYSIS

Duration of Exam – 3 hrs | ***Semester – II*** | ***Max. Marks – 70***

Part A (5x2= 10 marks)

Answer **ALL** questions

1. State Little wood second principle.
2. Define convergence in measure.
3. State Fatou's lemma.
4. Define algebra of sets with example.
5. Define p-integrable function.

Part B (5X5= 25 marks)

Answer **ALL** questions

6a. Prove that a constant function is integrable.

OR

6b. State $M_k - m_k$ theorem. Evaluate $\lim_{n\to\infty} \sup a_n$ and $\lim_{n\to\infty} \inf a_n$ where

$$a_n = \begin{cases} \frac{n}{n+1} & , if\ n\ is\ odd \\ \frac{1}{n+1} & , if\ n\ is\ even \end{cases}.$$

7a. Let μ be a measure on X. Then show that μ is countably subadditive.

OR

7b. Let $f: R \to R$ be given by $f(0) = 0$ and $f(x) = |x|, if\ x \neq 0$.Determine $(D^+f)(0), (D_+f)(0), (D^-f)(0)\ and\ (D_-f)(0)$.

8a. Define Function of bounded variation. Let $f: [0, 2] \to R$ given by $f(0) = 0$ and $f(x) = xsin\left(\frac{\pi}{x}\right), if x\ \neq 0$. Then show that f is not a function of bounded variation on $[0, 2]$.

OR

8b. Let f be a non negative measurable function and is integrable over a measurable set E. Let g be a nonnegative measurable function such that $g(x) < f(x),\ \forall x \in E$. Then g is also integrable over E and moreover $\int_E f - g = \int_E f - \int_E g$.

9a State and prove Little wood first principle.

OR

9b Define general Lebesgue integral and verify that $|f| = f^+ - f^-$.

10a Let $D \subseteq R$ be measurable. Let $f: D \to R$ be any continuous function then prove that f is measurable function.

OR

10b Let $\langle f_n \rangle$ be a sequence of real valued measurable functions defined on a measurable set E. Let f be a real valued measurable function defined on E such that $f_n \to f$ in measure on E. Then exists a subsequence $\left\langle f_{n_v} \right\rangle$ of $\langle f_n \rangle$ such that $f_{n_v} \to f$ almost everywhere on E.

Part C *(5X7= 35 marks)*

Answer **ALL** questions.

11a. State and prove Holders inequality.

OR

11b. Let f, g be measurable functions which are integrable over a measurable set E then prove that
$f + g$ is integrable over E and $\int_E f + g = \int_E f + \int_E g$.

12a. The countable union of countable sets is countable.

OR

12b. Let B be the collection of all Borel sets in R then prove that B is also the smallest σ-algebra of sets on R containing the collection of all closed sets in R.

13a. State and prove Minkowski inequality for $1 \leq p \leq \infty$.

OR

13b. Define upper and lower sum and show that both these sums are bounded and has supremum and infimum.

14a. Let $f: [a, b] \to R$ be a function of bounded variation on $[a, b]$ then prove that
$P - N = f(b) - f(a) \; and \; T = P + N.$

OR

14b. State and prove refinement lemma.

15a. Let $D \in \boldsymbol{M}$ and f be an extended real valued function defined on D then Prove that the following statements are equivalent:

i) $\{x \in D | f(x) > \alpha, \alpha \in R\}$ Is measurable.
ii) $\{x \in D | f(x) < \alpha, \alpha \in R\}$ Is measurable.
iii) $\{x \in D | f(x) \leq \alpha, \alpha \in R\}$ Is measurable.
iv) $\{x \in D | f(x) \geq \alpha, \alpha \in R\}$ Is measurable.

OR

15b. Define Lebesgue outer measure and prove that m^* is translation invariant.